

# On the relation between Bell inequalities and nonlocal games

J. Silman, S. Machnes, and N. Aharon

*School of Physics and Astronomy, Raymond and Beverly Sackler  
Faculty of Exact Sciences, Tel-Aviv University, Tel-Aviv 69978, Israel*

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**We investigate the relation between Bell inequalities and nonlocal games by presenting a systematic method for their bilateral conversion. In particular, we show that while to any nonlocal game there naturally corresponds a unique Bell inequality, the converse is not true. As an illustration of the method we present a number of nonlocal games that admit better odds when played using quantum resources.**

## I. INTRODUCTION

Quantum mechanics admits stronger correlations between remote parties than allowed by any (causal) classical theory [1]. These correlations, arising from the entanglement properties of the product Hilbert space, can be used in various nonlocal games [2] to improve on the maximum winning probability that obtains using only classical correlations [3, 4, 5, 6, 7, 8]. As such, these examples also constitute proofs of the nonlocal nature of quantum mechanics.

In this paper we explore the relation between nonlocal games, as defined by Cleve *et al.* [2], and Bell-type [1, 9] and GHZ-type [10] nonlocality proofs. In particular, we show that to any member of a certain class of Bell inequalities there “naturally” corresponds a nonlocal game. To complement this we show the converse as well; any nonlocal game can be uniquely mapped to a Bell inequality in this certain class.

## II. BACKGROUND

### A. Nonlocal games

As defined in [2], a nonlocal game is a cooperative task for a team of several remote players. Every player is randomly assigned by a verifier an input according to some joint probability distribution. Each then chooses one out of a set of possible outputs and sends it to the verifier. The verifier consults a truth table dictating for each combination of inputs, what combinations of outputs result in a win. The players know the winning conditions, as well as the joint probability distribution governing the assignment of combinations of inputs, and may coordinate a joint strategy prior to receiving them, but cannot communicate subsequently. A team making use of quantum correlations (shared entanglement) is said to employ a “quantum strategy”, whereas if not, is said to employ a “classical strategy”.

### B. Bell inequalities

In deterministic local hidden-variable theories all measurable quantities are predetermined. Locality enters via the requirement that the results of measurements carried out in any region are independent of what type of measurements were, are or will be carried out, if at all, in spacelike separated regions. In this light, let us consider two spacelike separated parties  $A$  and  $B$  sharing an entangled state of a pair of qubits. Suppose now that  $A$  measures the spin component of his qubit along some axis. The result that obtains must be independent of the axis along which  $B$  measures. This together with predeterminedism (realism) implies the following for two-spin measurement settings per party

$$a_1b_1 + a_1b_2 + a_2b_1 - a_2b_2 = \pm 2, \quad (1)$$

where  $a_i$  and  $b_i$  denote the value of the spin component of  $A$ 's qubit and  $B$ 's qubit along the axes  $\mathbf{n}_A^{(i)}$  and  $\mathbf{n}_B^{(i)}$ , respectively ( $i = 1, 2$ ). Averaging over many repeats of the experiment, or what amounts the same thing, averaging over the hidden-variable distribution, we obtain the CHSH inequality

$$|\langle a_1b_1 \rangle + \langle a_1b_2 \rangle + \langle a_2b_1 \rangle - \langle a_2b_2 \rangle| \leq 2, \quad (2)$$

or put differently,

$$1 \leq |P(a_1b_1 = 1) + P(a_1b_2 = 1) + P(a_2b_1 = 1) + P(a_2b_2 = -1)| \leq 3. \quad (3)$$

Now eq. (1) is nothing more than algebraic relation for two pairs of independent variables which assume the values  $\pm 1$ . Indeed, analogous relations exist for any number  $n$  of such pairs of variables

$$\sum_s c_s \prod_{i=1}^n o_i^{(s_i)} = -C, \dots, C, \quad (4)$$

where  $o_i^{(s_i)} = \pm 1$  and  $s$  is an  $n$ -component vector with  $s_i = 1, 2$ , the summation carried out over all possible vectors. And give rise to a whole host of Bell inequalities for “full” correlation functions of dichotomic outcomes (i.e. Bell inequalities that only consider the events

$\prod_{i=1}^n o_i^{(s_i)} = \pm 1$  with  $n$  the number of parties)

$$\left| \sum_s c_s \left\langle \prod_{i=1}^n o_i^{(s_i)} \right\rangle \right| \leq C, \quad (5)$$

or alternately, as weighted sums

$$S_C^{min} \leq \sum_s w_s P(O_s = \ell_s) \leq S_C^{max}, \quad \ell_s = \pm 1, \quad (6)$$

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$$\left\langle \prod_{i=1}^n o_i^{(s_i)} \right\rangle = P\left(\prod_{i=1}^n o_i^{(s_i)} = 1\right) - P\left(\prod_{i=1}^n o_i^{(s_i)} = -1\right) = 2P\left(\prod_{i=1}^n o_i^{(s_i)} = 1\right) - 1 = 1 - 2P\left(\prod_{i=1}^n o_i^{(s_i)} = -1\right), \quad (7)$$


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to substitute  $2P(\prod_{i=1}^n o_i^{(s_i)} = 1) - 1$  and  $1 - 2P(\prod_{i=1}^n o_i^{(s_i)} = -1)$  for  $\langle \prod_{i=1}^n o_i^{(s_i)} \rangle$  whenever  $c_s$  is positive or negative, respectively, and rearranging terms, we obtain

$$w_s = 2|c_s|, \quad S_C^{min} = C - \sum_s |c_s|, \quad S_C^{max} = C + \sum_s |c_s|. \quad (8)$$

Most generally, we may consider  $n$  spacelike separated parties, where the  $i$ -th party may measure  $m_i$  different observables, corresponding to different settings of their measurement device. Note that  $m_i$  need not equal  $m_j$ . Moreover, the different measurement settings employed by the same party need not have the same number of distinct outcomes. Any Bell inequality pertaining to this system admits a weighted average representation as follows

$$S_C^{min} \leq \sum_M w_M P(o_1^{(s_1)} = \lambda_1^{(s_1, r_1)}, \dots, o_n^{(s_n)} = \lambda_n^{(s_n, r_n)}) \leq S_C^{max}. \quad (9)$$

Here, given that player  $i$  employs the measurement setting  $s_i$ ,  $r_i$  labels the different possible outcomes.  $P(o_1^{(s_1)} = \lambda_1^{(s_1, r_1)}, \dots, o_n^{(s_n)} = \lambda_n^{(s_n, r_n)})$  is the probability that party 1 obtain the result  $\lambda_1^{(s_1, r_1)}$  when employing the measurement setting  $s_1$  ( $s_1 = 1, \dots, m_1$ ), party 2 obtain the result  $\lambda_2^{(s_2, r_2)}$  when employing the measurement setting  $s_2$  ( $s_2 = 1, \dots, m_2$ ), etc.  $M$  is a vector of ordered pairs  $(s_i, r_i)$ . Note that the summation is carried out over all possible  $M$ , i.e. all possible measurement settings and outcomes.

### III. FROM BELL INEQUALITIES TO NONLOCAL GAMES

Before presenting the method for converting Bell inequalities to nonlocal games *à la* Cleve *et al.* [2], we would like to motivate it on an intuitive level. Roughly speaking, the equivalence between the pair hinges on

The  $c_s$  and  $C$ , and the  $w_s$ ,  $S_C^{min}$  and  $S_C^{max}$  are of course related, and may easily be obtained from one another. For example, suppose we are given any one of the inequalities eq. (5). Using the identities

two key points of similarity: (i) In nonlocal games each player must choose his output without knowing the input assigned to any of the others. Similarly, in local hidden-variable theories the value of a physical quantity measured in one region obtains independently of which physical quantities were, are, or will be measured, if at all, in spacelike separated regions. (ii) There are no classical nonlocal game strategies that only allow for correct or preferable combinations of outputs (unless the game is trivial). In a like manner, local hidden-variable theories never saturate the algebraic limit of (nontrivial) Bell inequalities.

Consider the family of Bell inequalities, eq. (5). The first step in converting these into games is to suitably reinterpret the expectation values in this new context. To do so we present the concise Bell inequalities - nonlocal games dictionary. In the “language” of nonlocal games  $s_i$  denotes the input received by player  $i$ , while  $o_i^{(s_i)}$  represents his output.  $\langle \prod_{i=1}^n o_i^{(s_i)} \rangle$  is therefore the expectation value of the product of outputs given the set of inputs  $s$ . The hidden-variable indicates the choice of strategy, and the averaging is understood to be carried out with respect to the different strategies employed [11].

Next we need to introduce joint probability distributions to govern the assignment of inputs and truth tables. To this end, let us shift our attention to the equivalent formulation of these Bell inequalities, eq. (6). These relations hold for independent sets of dichotomic variables, whether these variables describe physical quantities or outputs in a nonlocal game. However, for the sums in these relations to make sense in the context of nonlocal games, we have to give meaning to the  $w_s$ . This is easily achieved by normalization, that is, we set the joint probability distribution for the inputs such that

$$\varrho_s = \frac{w_s}{\sum_s w_s} = \frac{|c_s|}{\sum_s |c_s|}. \quad (10)$$

If we now construct the truth tables such that the games

are considered to have been won iff

$$\prod_{i=1}^n o_i^{(s_i)} = \ell_s \triangleq \begin{cases} +1 & c_s > 0 \\ -1 & c_s < 0 \end{cases}, \quad (11)$$

where  $o_i^{(s_i)} = \pm 1$  is the output of player  $i$ , the superscript  $s_i$  serving to denote its (possible) dependence on the input, then the games' total winning probabilities are given by [12]

$$P_C^{min} \leq \sum_s \varrho_s P\left(\prod_{i=1}^n o_i^{(s_i)} = \ell_s\right) \leq P_C^{max}, \quad (12)$$

with

$$P_C^{max/min} = \frac{S_C^{max/min}}{\sum_s w_s} = \frac{C \pm \sum_s |c_s|}{2 \sum_s |c_s|}. \quad (13)$$

the maximum and minimum classical total winning probabilities.

Using a quantum strategy the classical maximum total winning probability can be surpassed. To do so the players must share a suitable entangled state. Upon receiving his input, each player measures the spin component of his qubit in a direction such that over many repetitions of the game a maximal violation of the originating Bell inequality would obtain. The maximum total winning probability is therefore given by

$$P_Q^{max} = \frac{S_Q^{max}}{\sum_s w_s} = \frac{Q + \sum_s |c_s|}{2 \sum_s |c_s|}, \quad (14)$$

where  $Q$  and  $S_Q^{max}$  denote the upper bounds imposed by quantum mechanics on the sums in eqs. (5) and (6), respectively. This gives an advantage of

$$\frac{Q - C}{2 \sum_s |c_s|} = \frac{S_Q^{max} - S_C^{max}}{\sum_s w_s} \quad (15)$$

to the optimal quantum strategy over the optimal classical one.

Bell inequalities for full correlation functions of dichotomic outcomes are part of a larger class of Bell inequalities, which have in common that in their weighted sum form, eq. (9), nonvanishing coefficients,  $w_M$ , pertaining to the same measurement settings are equal, and therefore independent of the outcome. Any member of this class can be converted into a nonlocal game. To see this we note that as weighted sums, eq. (9), these inequalities admit a simplified form

$$\begin{aligned} S_C^{min} &\leq \sum_s w_s \sum_{\mu} P(o_1^{(s_1)} = \lambda_1^{(s_1, \mu)}, \dots, o_n^{(s_n)} = \lambda_n^{(s_n, \mu)}) \\ &\leq S_C^{max}. \end{aligned} \quad (16)$$

Here the summation over  $\mu$  is carried out over different sets of outcomes, which are not necessarily mutually exclusive. That is,  $\lambda_k^{(s_k, \nu \neq \mu)}$  may equal  $\lambda_k^{(s_k, \mu)}$ .

$P(o_1^{(s_1)} = \lambda_1^{(s_1, \mu)}, \dots, o_n^{(s_n)} = \lambda_n^{(s_n, \mu)})$  is the probability that party 1 obtain the result  $\lambda_1^{(s_1, \mu)}$  when employing the measurement setting  $s_1$  ( $s_1 = 1, \dots, m_1$ ), party 2 obtain the result  $\lambda_2^{(s_2, \mu)}$  when employing the measurement setting  $s_2$  ( $s_2 = 1, \dots, m_2$ ), etc. The construction of the joint probability distribution governing the assignment of inputs and the truth table is analogous to that of the full correlation functions case. The joint probability distribution for the inputs is still obtained via eq. (10). However, the winning conditions can no longer be expressed by eq. (11). If up to normalization eq. (16) is to represent the game's total winning probability, then given a combination of inputs  $s$  the full set of winning combinations of outputs must equal  $\cup_{\mu} \{\lambda_1^{(s_1, \mu)}, \dots, \lambda_n^{(s_n, \mu)}\}$ . The game is then considered to have been won iff

$$\{o_1^{(s_1)}, \dots, o_n^{(s_n)}\} \subseteq \cup_{\mu} \{\lambda_1^{(s_1, \mu)}, \dots, \lambda_n^{(s_n, \mu)}\}, \quad (17)$$

where  $\{o_1^{(s_1)}, \dots, o_n^{(s_n)}\}$  denotes some combination of outputs returned by the players.

#### IV. FROM NONLOCAL GAMES TO BELL INEQUALITIES

When considering Bell inequalities for full correlation functions, up to normalization, the conversion gives rise to a one to one mapping between the coefficients of the Bell inequality, the  $w_s$ , and those of the input frequencies of the nonlocal game, the  $\varrho_s$ . See eq. (10). (That the mapping is not one to one between the  $c_s$  and the  $\varrho_s$ , is merely due to the fact that the Bell inequalities, eq. (5), remain unchanged if we flip the signs of all the  $c_s$ .) It is therefore straightforward to invert this procedure and use it to obtain a Bell inequality for full correlation functions from any nonlocal game with dichotomic outputs.

This one to one character of the mapping carries over to the conversion of any of the inequalities, eq. (16). (See the last paragraph in the previous subsection.) This leads to the conclusion that any nonlocal game can be converted into a Bell inequality.

#### V. EXAMPLES

We now give two examples illustrating the application of our method. In the first example we convert a family of Bell inequalities for full correlation functions into a corresponding family of nonlocal games. In the second example we illustrate the more general case of non-full correlation Bell inequalities.

##### A. Example I

We consider the following family of two-qubit Bell inequalities for  $n \times n$  measurement settings introduced by

Gisin [13]

$$\begin{pmatrix} 1 & \cdots & 1 & -1 \\ & \ddots & 1 & -1 \\ \vdots & & 1 & -1 \\ & 1 & -1 & \ddots \\ 1 & -1 & & \ddots & -1 \\ -1 & & \cdots & -1 & -1 \end{pmatrix} \leq \begin{cases} \frac{1}{2}n^2 & \text{even } n \\ \frac{1}{2}(n^2 + 1) & \text{odd } n \end{cases}. \quad (18)$$

Here the matrix's dimension is  $n \times n$  and its  $(i, j)$ -th component denotes the coefficient of  $\langle a_i b_j \rangle$ . For example, in this notation the CHSH inequality reads

$$\begin{pmatrix} 1 & -1 \\ -1 & -1 \end{pmatrix} \leq 2. \quad (19)$$

Since all the  $c_{ij}$  equal  $\pm 1$  it follows from eq. (10) that the joint probability distribution should be set as uniform

$$\varrho_{ij} = \frac{1}{n^2}. \quad (20)$$

As for the truth table, eq. (11) instructs us to require that identical (opposite) outputs be returned given the inputs  $i$  and  $j$  if the coefficient of  $\langle a_i b_j \rangle$  is positive (negative). Examining eq. (18), we see that the matrix's component are arranged such that

$$c_{ij} = \begin{cases} +1 & i+j \leq n \\ -1 & i+j > n \end{cases}. \quad (21)$$

The winning conditions therefore amount to the return of anticorrelated outputs given inputs whose sum is greater than  $n$ , and correlated otherwise. From eqs. (12) and (13) we then have

$$\begin{aligned} & \sum_{i+j \leq n} P(o_A^{(i)} = o_B^{(j)}) + \sum_{i+j > n} P(o_A^{(i)} = -o_B^{(j)}) \\ & \leq \begin{cases} \frac{3}{4} & \text{even } n \\ \frac{3}{4} + \frac{1}{4n^2} & \text{odd } n \end{cases}. \end{aligned} \quad (22)$$

We see that as  $n \rightarrow \infty$  the maximum total winning probability converges to 75%. In this limit we can effect a transition to the continuum. Introducing the variables

$$\alpha = \lim_{n \rightarrow \infty} \frac{i}{n}, \quad \beta = \lim_{n \rightarrow \infty} \frac{j}{n}, \quad (23)$$

the game translates to the task of returning identical outputs whenever  $\alpha + \beta \leq 1$ , and opposite outputs otherwise [14].

Higher probabilities can be reached using a quantum strategy. The maximum obtains when the players share a singlet state, with one of the players measuring at an angle of  $\frac{i}{\pi n}$  spanning from, say, the negative  $x$ -axis in the  $xy$ -plane, and the other at an angle of  $\frac{j}{\pi n}$  spanning from the negative  $y$ -axis in the same plane. The dependence of the maximum on  $n$  is given by

$$P_Q^{max} = \frac{\cos(\frac{\pi}{2n})}{n \sin(\frac{\pi}{n})} + \frac{1}{2}, \quad n \neq 1, \quad (24)$$

as is easily verified making use of eq. (14), with  $S_Q^{max}$  taken from [13]. Once again we see that as  $n \rightarrow \infty$  the maximum converges to a fixed value of  $\simeq 81.8\%$ .

## B. Example II

We consider the following Bell inequality for three qutrits [15]

$$\begin{aligned} & \left| P(o_A^{(1)} + o_B^{(1)} + o_C^{(1)} = 0) - P(o_A^{(1)} + o_B^{(1)} + o_C^{(2)} = 2) - P(o_A^{(1)} + o_B^{(2)} + o_C^{(1)} = 2) + P(o_A^{(1)} + o_B^{(2)} + o_C^{(2)} = 1) \right. \\ & \left. - P(o_A^{(2)} + o_B^{(1)} + o_C^{(1)} = 2) + P(o_A^{(2)} + o_B^{(1)} + o_C^{(2)} = 1) + P(o_A^{(2)} + o_B^{(2)} + o_C^{(1)} = 1) + 2P(o_A^{(2)} + o_B^{(2)} + o_C^{(2)} = 0) \right| \leq 3, \end{aligned} \quad (25)$$

where  $o_A^{(i)}, o_B^{(j)} = 1, 0, -1$  and all the equalities are evaluated modulus three. Substituting

$$P(o_A^{(i)} + o_B^{(j)} + o_C^{(k)} = n) = 1 - P(o_A^{(i)} + o_B^{(j)} + o_C^{(k)} \neq n) = 1 - P(o_A^{(i)} + o_B^{(j)} + o_C^{(k)} = n+1) - P(o_A^{(i)} + o_B^{(j)} + o_C^{(k)} = n-1) \quad (26)$$

for each of the probabilities in the second line of eq. (25) with we get

$$0 \leq \beta \leq 6, \quad (27)$$

$$\begin{aligned} \beta \hat{=} & P(o_A^{(1)} + o_B^{(1)} + o_C^{(1)} = 0) + P(o_A^{(1)} + o_B^{(1)} + o_C^{(2)} \neq 2) + P(o_A^{(1)} + o_B^{(2)} + o_C^{(1)} \neq 2) + P(o_A^{(1)} + o_B^{(2)} + o_C^{(2)} = 1) \\ & + P(o_A^{(2)} + o_B^{(1)} + o_C^{(1)} \neq 2) + P(o_A^{(2)} + o_B^{(1)} + o_C^{(2)} = 1) + P(o_A^{(2)} + o_B^{(2)} + o_C^{(1)} = 1) + 2P(o_A^{(2)} + o_B^{(2)} + o_C^{(2)} = 0). \end{aligned} \quad (28)$$

Eq. (10) now instructs us to set the joint probability for the inputs,  $i, j, k = 1, 2$ , as follows

$$\varrho_{ijk} = \frac{1}{9} (1 + \delta_{i,2} \delta_{j,2} \delta_{k,2}). \quad (29)$$

While from eq. (17) we have that given  $i = j = k = 1$  outputs satisfying  $o_A^{(1)} + o_B^{(1)} + o_C^{(1)} = 0$  must be returned, given  $i = j = k = 2$  outputs satisfying  $o_A^{(1)} + o_B^{(1)} + o_C^{(2)} \neq 2$  must be returned, etc. The maximum classical total winning probability is then  $\simeq 66.7\%$ . See eq. (13).

From [15] we numerically have that  $S_Q^{max} \simeq 7.37$ . The maximum quantum total winning probability is therefore  $\simeq 81.9\%$ , resulting in a  $\simeq 15.2\%$  quantum advantage.

## VI. CONCLUSION

To conclude, we have presented a systematic method for the bilateral conversion of any of the Bell inequalities, eq. (16), into nonlocal games. In particular, previously introduced nonlocal games are all seen to share a common thread in this unified approach. The method is not applicable to Bell inequalities which cannot assume a form as in eq. (16), because for each of these at least one of the measurement settings admits unequal nonvanishing coefficients  $w_M$ , eq. (9). This of course does not mean that another method cannot be devised to convert any

Bell inequality into a nonlocal game. However, it seems very likely that such an increase in generality must come at the expense of the one to one property of the mapping between the two; a nonlocal game would then no longer fully encapsulate the unique character of the originating Bell inequality [16].

In this context the work of Brukner *et al.*, who showed that to every Bell inequality there corresponds a communication complexity problem (CCP) [17], should be mentioned (see also [18]). Indeed, any nonlocal game can be cast as a CCP. Nevertheless, no conflict arises with our previous conclusion, as not every CCP can be cast as a nonlocal game (as defined in [2]).

Recently, it has been argued that nonlocal games may be used to devise loop-hole free experimental tests of local realism [3, 6]. To this end, we hope that our method may prove useful. Moreover so, if there is indeed a price to be paid for generality.

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